NEET | IIT-JEE | FOUNDATION

JEE Advanced - 2021

Paper-II

(Held on 3rd October, 2021)

TEST PAPER
with
SOLUTIONS

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PART A : PHYSICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

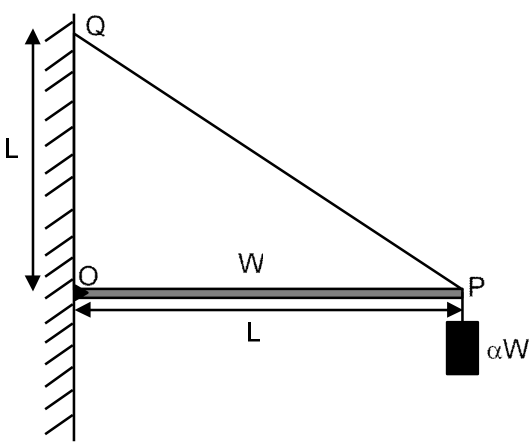
  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of ($2\sqrt{2}$)W. Which of the following statement(s) is(are) correct ?



(A) The vertical component of reaction force at O does not depend on α

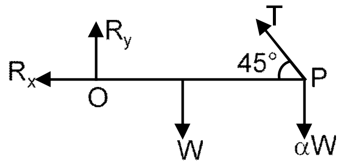
(B) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

(C) The tension in the rope is 2W for $\alpha = 0.5$

(D) The rope breaks if $\alpha > 1.5$

Ans. (A, B, D)

Sol.



$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \qquad ....(i)$$

$$R_x = \frac{T}{\sqrt{2}} \qquad ....(ii)$$

Taking torque about 'O'

$$W\frac{\ell}{2} + \alpha W \ell = \frac{T}{\sqrt{2}}\ell$$

$$T = \sqrt{2}\left(\frac{W}{2} + \alpha W\right) \qquad ....(iii)$$

$$R_x = \frac{T}{\sqrt{2}} = \left(\frac{W}{2} + \alpha W\right)$$

Taking torque about P

$$R_y \ell = W\frac{\ell}{2}$$

$$R_y = \frac{W}{2};$$

when $T = T_{max}$

$$2\sqrt{2}W = \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$$

we get $\alpha = \frac{3}{2}$

2. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance ?

(A) $u = 0.8\ v$ and $f_s = f_0$

(B) $u = 0.8\ v$ and $f_s = 2f_0$

(C) $u = 0.8\ v$ and $f_s = 0.5\ f_0$

(D) $u = 0.5\ v$ and $f_s = 1.5\ f_0$

Ans. (A, D)

Sol. $f = f_s\left(\dfrac{v}{v-u}\right)$

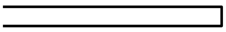
(A) $f = f_0\left(\dfrac{v}{v-0.8v}\right) = 5f_0$

(B) $f = 2f_0\left(\dfrac{v}{v-0.8v}\right) = 10f_0$

(C) $f = 0.5f_0\left(\dfrac{v}{v-0.8v}\right) = 2.5f_0$

(D) $f = 1.5f_0\left(\dfrac{v}{v-0.5v}\right) = 3f_0$

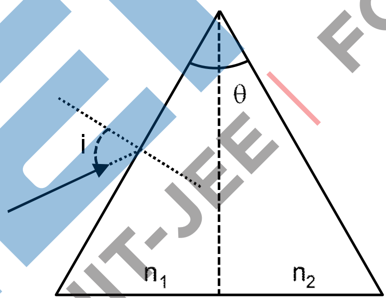
close



All odd harmonics are available in closed pipe therefore

Correct Ans (A, D)

3. For a prism of prism angle $\theta = 60^\circ$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is (are) correct ?



(A) The value of Δe (in radians) is greater than that of Δn

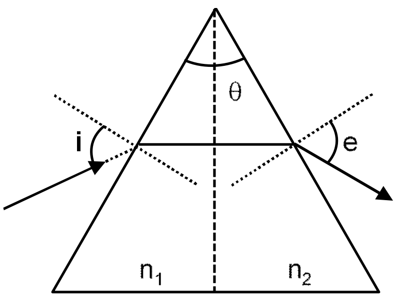
(B) Δe is proportional to Δn

(C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

(D) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

Ans. (B, C)

Sol.



$$1\times \sin i=\mu \sin\left(\frac{A}{2}\right)$$

$$\sin i=\frac{3}{4}$$

n_1 sin30° = 1sin(e)

on differentiating both sides

dn sin30° = de cos(e)

$$de=\frac{dn}{2\cos(e)}$$

$$=\frac{dn}{2\times\frac{\sqrt{7}}{4}}$$

$$de=\frac{2}{\sqrt{7}}dn \Rightarrow de < dn$$

$$de=\frac{2.8\times10^{-3}\times2}{\sqrt{7}}=2.11\,\text{mrad}$$

4. A physical quantity $\vec{S}$ is defined as $\vec{S}=(\vec{E}\times\vec{B})\,/\,\mu_0$, where E is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies) ?

(A) $\dfrac{\text{Energy}}{\text{charge}\times\text{current}}$ (B) $\dfrac{\text{Force}}{\text{Length}\times\text{Time}}$ (C) $\dfrac{\text{Energy}}{\text{Volume}}$ (D) $\dfrac{\text{Power}}{\text{Area}}$

Ans. (B, D)

Sol. $\vec{S}=[\vec{E}\times\vec{B}]\dfrac{1}{\mu_0}$

S is pointing vector denotes flow of energy per unit area per unit time

$$\vec{S}=\frac{\text{watt}}{m^2}$$

Hence B, D are correct

5. A heavy nucleus N, at rest, undergoes fission N $\rightarrow$ P + Q, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speed of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct ?

(A) $E_P + E_Q = c^2\,\delta$

(B) $E_P=\left(\dfrac{M_P}{M_P-M_Q}\right)c^2\delta$

(C) $\dfrac{v_P}{v_Q}=\dfrac{M_Q}{M_P}$

(D) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \frac{M_P M_Q}{(M_P + M_Q)}$

Ans. (A, C, D)

Sol. $N \longrightarrow P + Q$

Energy released = $(m_N - m_P - m_Q)c^2 = \delta c^2$

This will be distributed kinetic energy of P and Q

$\Rightarrow E_P + E_q = \delta c^2$ ....(i)

By conservation of momentum

$$v_p = \frac{p}{m_p} \longleftarrow \underset{m_p}{P} \qquad \underset{m_q}{Q} \longrightarrow \frac{p}{m_N} = v_Q$$

So $\frac{v_p}{v_q} = \frac{M_q}{M_p}$ .....(ii)

Kinetic energy be written as $KE = \frac{p^2}{2m}$

Hence divided in inverse ratio of masses.

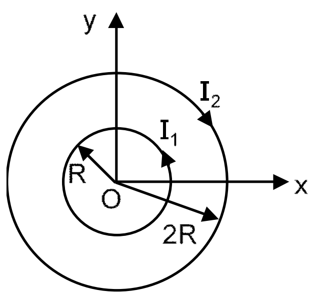
$E_p = \frac{M_q}{M_p + M_q} c^2\delta$ ....(iii)

By equation (i) $\Rightarrow \frac{p^2}{2M_p} + \frac{p^2}{2M_q} = \delta c^2$

$\Rightarrow \frac{p^2}{2\mu} = \delta c^2 \Rightarrow p = c\sqrt{2\mu\delta}$

Ans. (A, C, D)

6. Two concentric circular loops, one of radius R and the other of radius 2R, lie in the xy-plane with the origin as their common center, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) current?



(A) $\vec{B}(x,y)$ is perpendicular to the xy-plane at any point in the plane

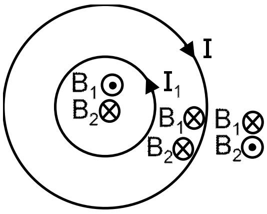
(B) $|\vec{B}(x,y)|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$

(C) $|\vec{B}(x,y)|$ is non-zero at all points for r < R

(D) $\vec{B}(x,y)$ points normally outward from the xy-plane for all the points between the two loops

Ans. (A, B)

Sol.



(A) $d\vec{B} = \frac{\mu_0 i d\vec{\ell} \times \vec{r}}{4\pi r^3}$

$\vec{d\ell}$ is in xy plane & $\vec{r}$ is also in xy plane

so $d\vec{B}$ is perpendicular to xy plane

(B) Due to symmetry it depends only on $r = \sqrt{x^2 + y^2}$

(C) At centre $B_1 = \frac{\mu_0 I_1}{2R}$; $B_2 = \frac{\mu_0 I_2}{2R} \Rightarrow B_2 > B_1$

but as we approach towards first loop B_1 increases to infinity hence B_1 dominates.

So it would be zero at some point between inner loops and centre.

Ans. (A, B)

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

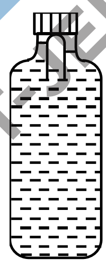
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $P_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.

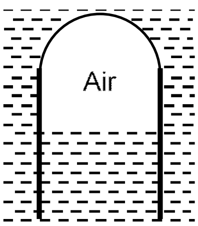
Let $\Delta v = X$ cc and $\Delta p = Y \times 10^3$ Pa.



7. The value of X is ____.

Ans. (0.30)

Sol.



When it starts sinking

$F_B = mg$

$\rho_0 (V_{glass} + V_{gas}) = m$

$1(2 + V_{gas}) = 5 \Rightarrow V_{gas} = 3cc$

Hence $\Delta V = 0.3$ cc.

8. The value of Y is ____.

Ans. (10.00)

Sol. Isothermal process for air

$P_1V_1 = P_2V_2$

$10^5 (3.3) = P_2 (3)$

$P_2 = 1.1 \times 10^5$

$\Delta P = P_2 - P_1 = 1.1 \times 10^5 - 10^5$

$= 0.1 \times 10^5$

$= 10 \times 10^3$ Pascal

$= Y \times 10^3$ Pascal

So, Y = 10

Question Stem for Question Nos. 9 and 10

Question Stem

A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L = 1.0 m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m^2/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

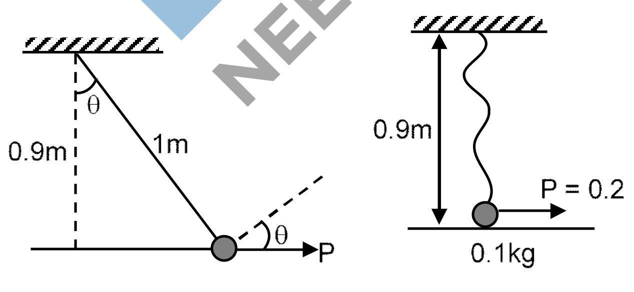
9. The value of J is ____.

Ans. (0.18)

10. The value of K is ____.

Ans. (0.16)

Sol.



$L = P \times 0.9 = 0.18$ kgm^2/s

Just after string becomes taut; there will be no velocity along the string.

$$\therefore V_{\perp} = \frac{P\cos\theta}{m} = \frac{0.2 \times 0.9}{1 \times 0.1} = 1.8 \text{ m/s}$$

$\therefore K = \frac{1}{2}mv_{\perp}^2 = \frac{1}{2} \times 0.1 \times 1.8^2$

= 0.162 J

Question Stem for Question Nos. 11 and 12

Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ.

Assume, $\pi\sqrt{3} \approx 5$.

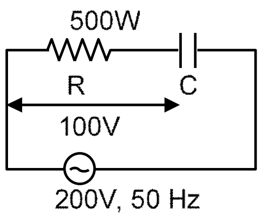
11. The value of C is ____.

Ans. (100.00)

12. The value of ϕ is ____.

Ans. (60.00)

Sol.



$\sqrt{V_C^2 + V_R^2} = \varepsilon_{rms}$

$\Rightarrow V_C^2 + 100^2 = 200^2$

$V_C = 100\sqrt{3}V$ ....(i)

$\tan\phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100}$

$\therefore \phi = 60°$ .....(ii)

$P = I_{rms}\, \varepsilon_{rms} \cos\phi = \frac{\varepsilon_{rms}^2}{z}\frac{1}{2}$

$500 = \frac{200}{z}\frac{1}{2}$

$\therefore z = 40\Omega$ ....(iii)

$\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{40}$

$\therefore R = 20$

& $x_C = \sqrt{Z^2 - R^2} = \sqrt{40^2 - 20^2} = 20\sqrt{3}\ \Omega$

$\Rightarrow \frac{1}{\omega C} = 20\sqrt{3}$ $\therefore C = 100$

SECTION-3 : (Maximum Marks : 12)

- This section contains **TWO (02) paragraphs**. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

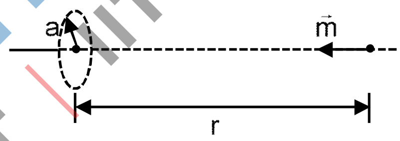
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r (>> a) from the center of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\frac{\mu_0 m}{2\pi r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



13. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

(A) $\frac{m}{r^3}$ (B) $\frac{m^2}{r^2}$ (C) $\frac{m}{r^2}$ (D) $\frac{m^2}{r}$

Ans. (A)

14. The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to

(A) $\frac{m}{r^5}$ (B) $\frac{m^2}{r^5}$ (C) $\frac{m^2}{r^6}$ (D) $\frac{m^2}{r^7}$

Ans. (C)

Sol. $\phi = Li = \frac{\mu_0 m}{2\pi r^3} \times \pi a^2$

$\Rightarrow i = \frac{\mu_0 m \pi a^2}{2\pi r^3 L}$

$\Rightarrow i \propto \frac{m}{r^3}$

$m' = \pi a^2 i = \frac{\mu_0 m \pi^2 a^4}{2\pi r^3 L}$

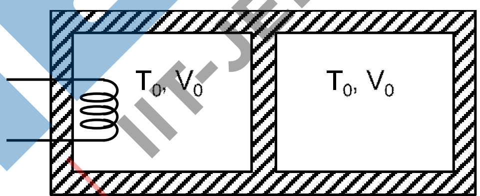
$F = \frac{k m^2 \pi^2 a^4}{2\pi r^7 L}$

$W = \int F\,dr \propto \int \frac{m^2\,dr}{r^7}$

$W \propto \frac{m^2}{r^6}$

Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_v = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.



15. The value of $\frac{T_R}{T_0}$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

Ans. (A)

16. The value of $\frac{Q}{RT_0}$ is

(A) $4(2\sqrt{2}+1)$ (B) $4(2\sqrt{2}-1)$ (C) $(5\sqrt{2}+1)$ (D) $(5\sqrt{2}-1)$

Ans. (B)

Sol. Finally $V_L = \frac{3V_0}{2}$, $V_R = \frac{V_0}{2}$

$$C_V = \frac{R}{\gamma - 1} = 2R \Rightarrow \gamma - 1 = \frac{1}{2}$$

$$\gamma = \frac{3}{2}$$

$$T_0 V_0^{\gamma-1} = T_R \left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

$$\rho\left(\frac{V_0}{2}\right)^{\gamma} = P_0 V_0^{\gamma} \Rightarrow P = P_0 \times 2^{\frac{3}{2}}$$

$$\frac{PV}{T_L} = \frac{P_0 V_0}{T_0} \Rightarrow T_L = 2^{\frac{3}{2}} \times \frac{3}{2} T_0 = 3\sqrt{2}\, T_0$$

$$Q = nC_V \Delta T_1 + nC_V \Delta T_2$$

$$= 1 \times 2R \times \left(3\sqrt{2} - 1\right) T_0 + 1 \times 2R \times \left(\sqrt{2} - 1\right) T_0$$

$$\frac{Q}{RT_0} = 2\left(3\sqrt{2} - 1\right) + 2\left(\sqrt{2} - 1\right) = 8\sqrt{2} - 4$$

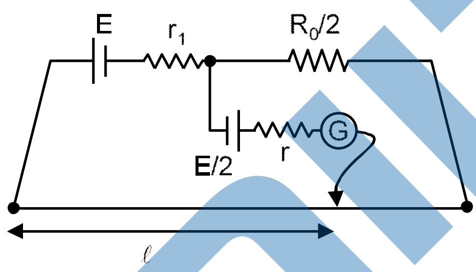
SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

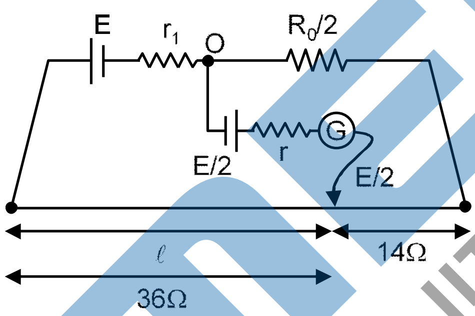
  Zero Marks : 0 In all other cases.

17. In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance $R_0 = 50\ \Omega$, a resistance $R_0/2$, another cell of emf E/2 (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at ℓ = 72 cm, then the value of r_1 =___Ω.



Ans. (3)

Sol.



$$i\left(\frac{R_0}{2}+0.28R_0\right)=\frac{E_0}{2}$$

$$i\times 0.78R_0=\frac{E_0}{2}$$

$$i=\frac{E_0}{2\times 0.78R_0}=\frac{E_0}{r_1+\frac{3}{2}R_0}$$

r_1 + 1.5 R_0 = 1.56 R_0

r_1 = 0.06 R_0

= 0.06 × 50 = 3 Ω

18. The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is ___.

Ans. (9)

Sol. Circular orbits

$$T = 2\pi\sqrt{\frac{R^2}{GM_S}}$$

Binary stars

$$nT = 2\pi\sqrt{\frac{(9R)^2}{G(3M_S + 6M_S)}}$$

$$n \times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^2}{GM_S}}$$

n = 9

19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R, respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is ___.

Ans. (6)

Sol. For P & Q

$E_1 - 4 = E_P$

$E_1 - 4.5 = E_Q$

$E_P = 2\,E_Q$

$E_1 - 4 = 2\,(E_1 - 4.5)$

$E_1 = 5$ eV

$E_P = 1$ eV, $E_Q = E_R = 0.5$ eV

For $E_2 - 5.5 = 0.5$

$E_2 = 6$ eV

PART B : CHEMISTRY

SECTION-1 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

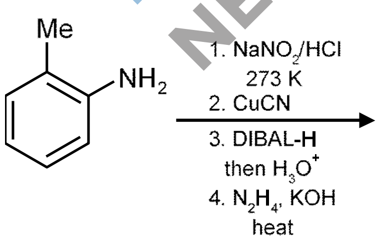
choosing ONLY (B) will get +1 mark;

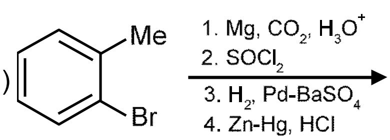
choosing ONLY (D) will get +1 mark;

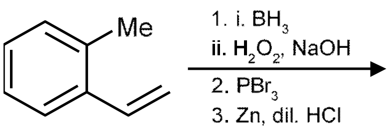
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

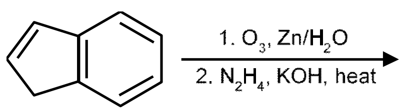
choosing any other option(s) will get −2 marks.

1. The reaction sequence(s) that would lead to o-xylene as the major product is (are)

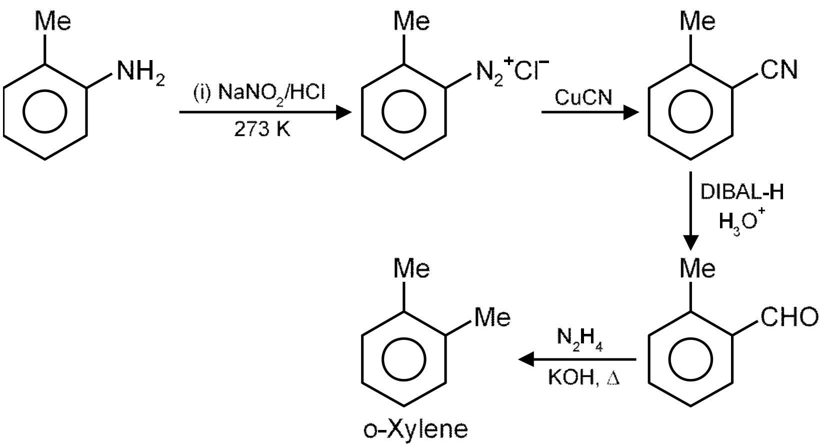
(A) 

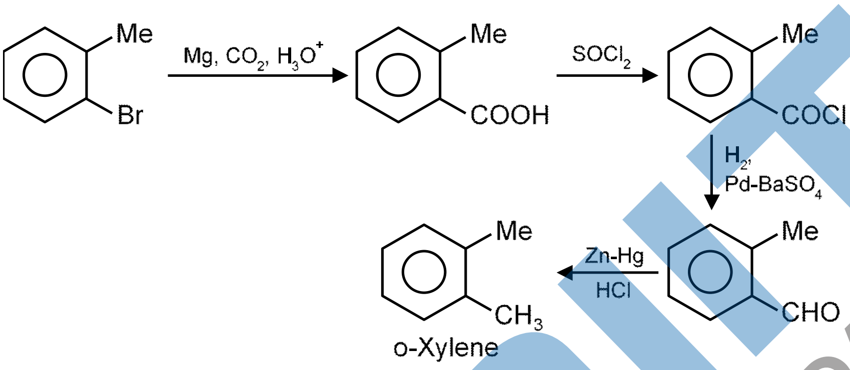
(B) 

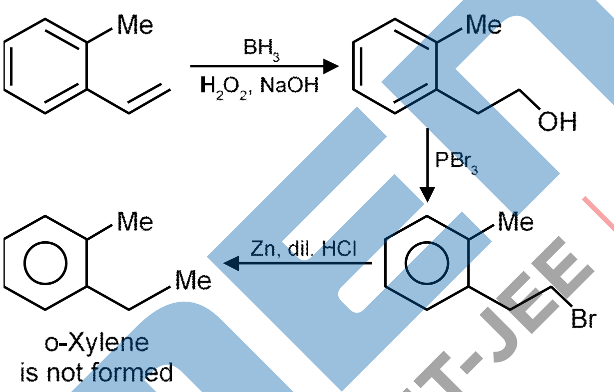
(C) 

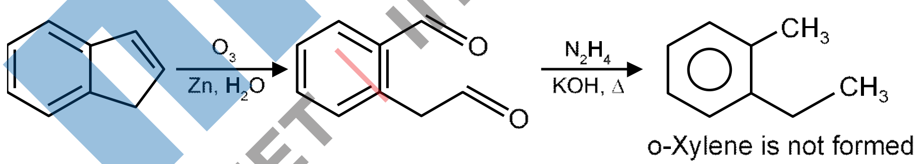
(D) 

Ans. (A, B)

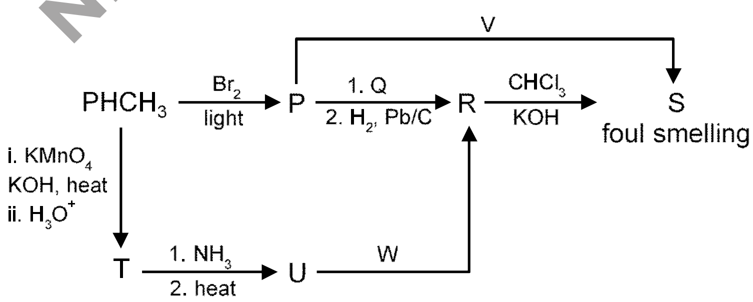
Sol. (A) 

(B) 

(C) 

(D) 

2. Correct option(s) for the following sequence of reactions is(are)



(A) Q = KNO_2, W = $LiAlH_4$

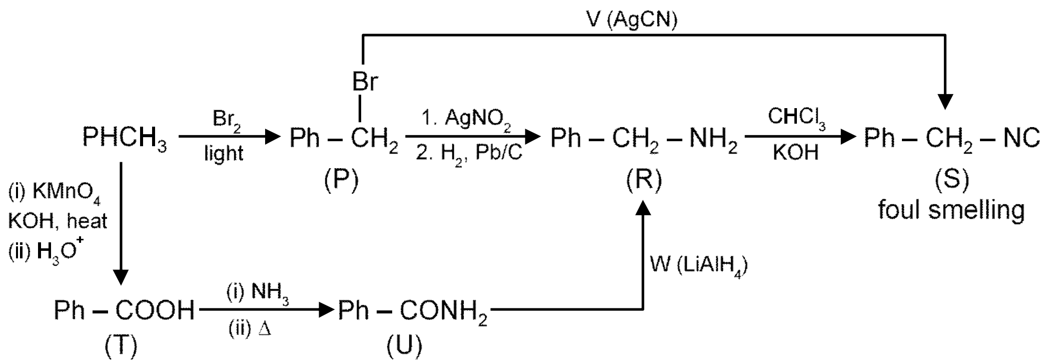
(B) R = benzenamine, V = KCN

(C) Q = $AgNO_2$, R = phenylmethanamine

(D) W = $LiAlH_4$, V = AgCN

Ans. (C, D)

Sol.



3. For the following reaction

$$2X + Y \xrightarrow{k} P$$

the rate of reaction is $\frac{d[P]}{dt} = k[X]$. Two moles of X are mixed with one mole of Y to make 1.0 L of solution. At 50 s, 0.5 mole of Y is left in the reaction mixture. The correct statement(s) about the reaction is(are)

(Use: ln 2 = 0.693)

(A) The rate constant, k, of the reaction is $13.86 \times 10^{-4}\ s^{-1}$.

(B) Half–life of X is 50s.

(C) At 50 s, $-\frac{d[X]}{dt} = 13.86 \times 10^{-3}$ mol L^{-1} s^{-1}.

(D) At 100 s, $-\frac{d[Y]}{dt} = 3.46 \times 10^{-3}$ mol L^{-1} s^{-1}.

Ans. (B, C, D)

Sol.

$$\frac{dp}{dt} = k[x]^1$$

	2x	+	y	→	p
t = 0	2		1		
t = 50 s	(2–1)		(1–0.5)		0.5

$$-\frac{1}{2}\frac{dx}{dt} = \frac{dp}{dt} = k[x]^1$$

$$-\frac{dx}{dt} = 2k[x]^1$$

$$2k = \frac{\ln 2}{50} \Rightarrow k = \frac{\ln 2}{100}$$

At $\quad 50 \text{ sec} \frac{-dx}{dt} = 2k \times (1)^1 = \frac{\ln 2}{50}$

At $\quad 100 \text{ sec} -\frac{1}{2}\frac{dx}{dt} = \frac{-dy}{dt} \Rightarrow -\frac{dy}{dt} = \frac{\ln 2}{100} \times \frac{1}{2}\left\{\frac{-dy}{dt} = k[x]^1\right\}$

4. Some standard electrode potentials at 298 K are given below:

Pb^{2+}/Pb	–0.13 V
Ni^{2+}/Ni	–0.24 V
Cd^{2+}/Cd	–0.40 V
Fe^{2+}/Fe	–0.44 V

To a solution containing 0.001 M of X^{2+} and 0.1 M of Y^{2+}, the metal rods X and Y are inserted (at 298 K) and connected by a conducting wire. This resulted in dissolution of X. The correct combination(s) of X and Y, respectively, is (are)

(Given: Gas constant, R = 8.314 J K^{-1} mol^{-1},

Faraday constant, F = 96500 C mol^{-1})

(A) Cd and Ni (B) Cd and Fe (C) Ni and Pb (D) Ni and Fe

Ans. (A, B, C)

Sol. $x(s) \longrightarrow x^{+2}$ (0.001 M) + $2e^-$ (anode)

y^{+2} (0.1 M) + $2e^- \longrightarrow y$ (s) (cathode)

$$E_{cell} = E^\circ_{cell} - \frac{0.06}{2}\log\frac{x^{+2}}{y^{+2}}$$

$E_{cell} = E^\circ_{cell} + 0.06$

(A) Cd and Ni $E^\circ_{cell} = +0.4 - 0.24$; $E_{cell} = 0.22$

(B) Cd and Fe $E^\circ_{cell} = -0.04$; $E_{cell} = 0.02$

(C) Ni and Pb $E^\circ_{cell} = 0.11$; $E_{cell} = 0.17$

(D) Ni and Fe $E^\circ_{cell} = -0.2$; $E_{cell} = -0.14$

since in (A) (B) (C) E_{cell} is positive hence answer is (A) (B) (C).

5. The pair(s) of complexes wherein both exhibit tetrahedral geometry is(are)

(Note: py = pyridine Given: Atomic numbers of Fe, Co, Ni and Cu are 26, 27, 28 and 29, respectively)

(A) $[FeCl_4]^-$ and $[Fe(CO)_4]^{2-}$ (B) $[Co(CO)_4]^-$ and $[CoCl_4]^{2-}$

(C) $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ (D) $[Cu(py)_4]^+$ and $[Cu(CN)_4]^{3-}$

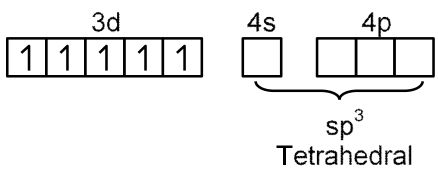
Ans. (A, B, D)

Sol. (A) $[FeCl_4]^-$

$Fe \rightarrow [Ar]\ 3d^6\ 4s^2$

$Fe^{+3} \rightarrow [Ar]\ 3d^5\ 4s^0$

Cl^- is W.F.L. and does not pair up the unpaired electron of central metal atom.

$\therefore$ Fe^{3+} (d^5) in $[FeCl_4]^-$ 

$[Fe(CO)_4]^{2-}$

$Fe \rightarrow [Ar]\ 3d^6\ 4s^2$

$Fe^{2-} \rightarrow [Ar]\ 3d^8\ 4s^2$

$\therefore Fe^{2-}\ (d^{10})$ in $[Fe(CO)_4]^{2-}$ $\overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}}$ $\underbrace{\overset{4s}{\boxed{}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}}_{sp^3}$

Tetrahedral

(B) $[Co(CO)_4]^-$

$Co \rightarrow [Ar]\ 3d^7\ 4s^2$

$Co^{-1} \rightarrow [Ar]\ 3d^8\ 4s^2$

$\therefore Co^-\ (d^{10})$ in $[Co(CO)_4]^-$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}$ $\underbrace{\overset{4s}{\boxed{}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}}_{sp^3}$

Tetrahedral

$[Co(CO)_4]^{2-}$

$Co \rightarrow [Ar]\ 3d^7\ 4s^2$

$Co^{+2} \rightarrow [Ar]\ 3d^7\ 4s^0$

Cl^- is W.F.L. and does not pair up the unpaired electron of central metal atom.

$\therefore Co^{2+}\ (d^7)$ in $[CoCl_4]^{2-}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ $\underbrace{\overset{4s}{\boxed{}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}}_{sp^3}$

Tetrahedral

(C) $[Ni(CO)_4]$

$Ni \rightarrow [Ar]\ 3d^8\ 4s^2$

$Ni^0 \rightarrow [Ar]\ 3d^8\ 4s^2$

$\therefore Ni\ (d^{10})$ in $[Ni(CO)_4]$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}$ $\underbrace{\overset{4s}{\boxed{}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}}_{sp^3}$

Tetrahedral

$[Ni(CN)_4]^{2-}$

$Ni \rightarrow [Ar]\ 3d^8\ 4s^2$

$Ni^{+2} \rightarrow [Ar]\ 3d^8\ 4s^0$

CN^- is S.F.L. and pair up the unpaired electron of central metal atom.

$\therefore Ni\ (d^8)$ in $[Ni(CN)_4]^{2-}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\underbrace{\boxed{}\ \overset{4s}{\boxed{}}\ \overset{4p}{\boxed{}\boxed{}\boxed{}}}_{dsp^3}$

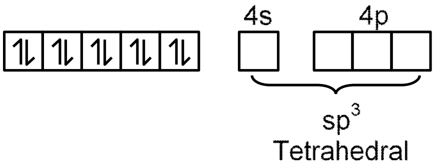
Square planar

(D) $[Cu(py)_4]^+$

Cu $\rightarrow$ [Ar] $3d^{10}\ 4s^{1}$

$Cu^{+1} \rightarrow$ [Ar] $3d^{10}\ 4s^{0}$

$\therefore$ Cu^{+1} (d^{10}) in $[Cu(py)_4]^{+}$



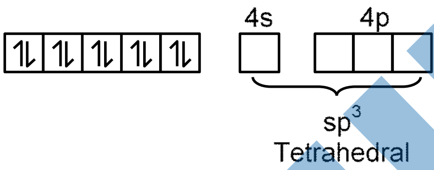
$[Cu(CN)_4]^{3-}$

Cu $\rightarrow$ [Ar] $3d^{10}\ 4s^{1}$

$Cu^{+1} \rightarrow$ [Ar] $3d^{10}\ 4s^{0}$

CN^{-} is S.F.L. and pair up the unpaired electron of central metal atom.

$\therefore$ Cu^{+1} (d^{10}) in $[Cu(CN)_4]^{+}$



6. The correct statement(s) related to oxoacids of phosphorous is(are)

(A) Upon heating, H_3PO_3 undergoes disproportionation reaction to produce H_3PO_4 and PH_3.

(B) While H_3PO_3 can act as reducing agent, H_3PO_4 cannot.

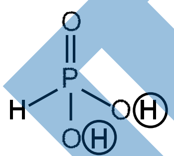
(C) H_3PO_3 is a monobasic acid.

(D) The H atom of P–H bond in H_3PO_3 is not ionizable in water.

Ans. (A, B, D)

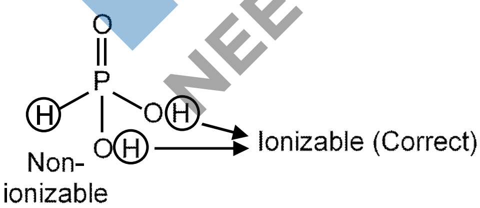
Sol. (A) $4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$ (correct)

(B) H_3PO_4 has "P" in its highest oxidation state, hence cannot act as a reducing agent (correct)

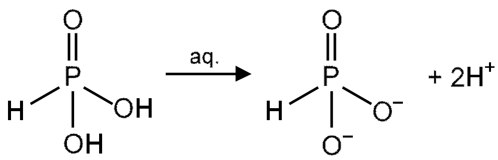
(C) 

Dibasic acid (incorrect)

Two OH group present in H_3PO_3

(D) 

The hydrogen which is directly attached to phosphorous does not ionized in water.



SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

At 298 K, the limiting molar conductivity of a weak monobasic acid is 4×10^2 S cm^2 mol^{-1}. At 298 K, for an aqueous solution of the acid the degree of dissociation of α and the molar conductivity is $y \times 10^2$ S cm^2 mol^{-1}. At 298 K, upon 20 times dilution with water, the molar conductivity of the solution becomes $3y \times 10^2$ S cm^2 mol^{-1}.

7. The value of α is ______.

Ans. (0.21 or 0.22)

Sol. $K_a = \dfrac{\Lambda_m^2 C}{\Lambda_m^\circ\left(\Lambda_m^\circ - \Lambda_m\right)}$

$$K_a = \frac{(y \times 10^2)^2 C}{4 \times 10^2 (4 \times 10^2 - y \times 10^2)} = \frac{(3y \times 10^2)^2 \times \frac{C}{20}}{4 \times 10^2 (4 \times 10^2 - 3y \times 10^2)}$$

$$\Rightarrow \frac{1}{(4-10)} = \frac{9}{20(4-3y)} \Rightarrow y = \frac{44}{51}$$

$$\alpha = \frac{\frac{44}{51} \times 10}{4 \times 10^2}$$

$\alpha = 0.2156$ ($\alpha = 0.22$ or 0.21)

$y = 0.86$

8. The value of y is ______.

Ans. (0.86)

Sol. $K_a = \dfrac{\Lambda_m^2 C}{\Lambda_m^\circ\left(\Lambda_m^\circ - \Lambda_m\right)}$

$$K_a = \frac{(y \times 10^2)^2 C}{4 \times 10^2 (4 \times 10^2 - y \times 10^2)} = \frac{(3y \times 10^2)^2 \times \frac{C}{20}}{4 \times 10^2 (4 \times 10^2 - 3y \times 10^2)}$$

$$\Rightarrow \frac{1}{(4-10)} = \frac{9}{20(4-3y)} \Rightarrow y = \frac{44}{51}$$

$$\alpha = \frac{\frac{44}{51} \times 10}{4 \times 10^2}$$

α = 0.2156 (α = 0.22 or 0.21)

y = 0.86

Question Stem for Question Nos. 9 and 10

Question Stem

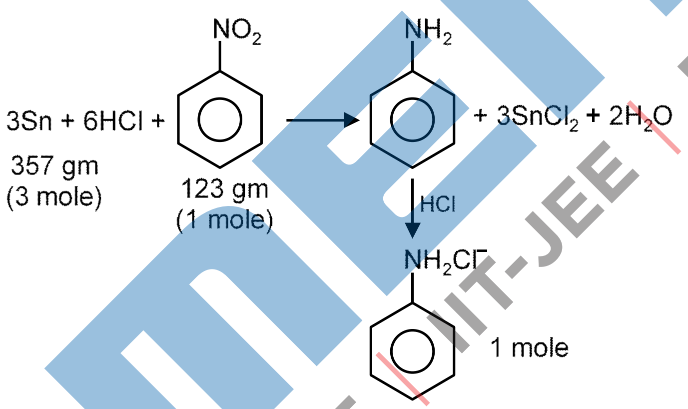
Reaction of x g of Sn with HCl quantitatively produced a salt. Entire amount of the salt reacted with y g of nitrobenzene in the presence of required amount of HCl to produce 1.29 g of an organic salt (quantitatively).

(Use Molar masses (in g mol^{-1}) of H, C, N, O, Cl and Sn as 1, 12, 14, 16, 35 and 119, respectively).

9. The value of x is ______.

Ans. (3.57)

Sol. The value of x is



(72 + 8 + 35) + 14

= 129gm (molecular weight of organic salt)

So to get 1.29 gm organic salt.

We have to form 0.01 mole salt.

So 0.01 mole nitrobenzene is required.

0.03 mole Sn is required.

So the amount of nitrobenzene = 0.01 × 123 = 1.23 gm

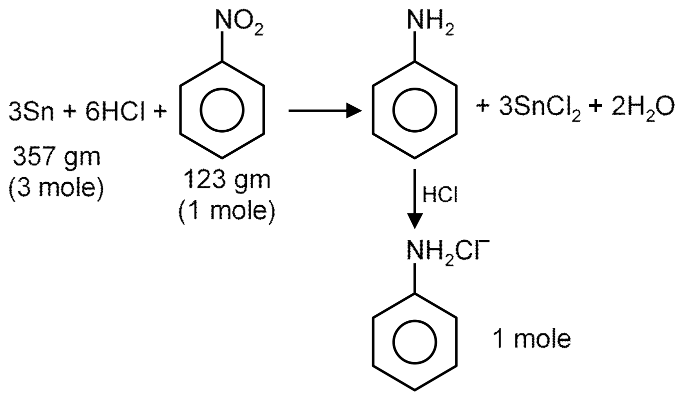
the amount of Sn required = 0.01 × 357 = 3.57 gm

Ans. 3.57 & 1.23

10. The value of y is ______.

Ans. (1.23)

Sol. The value of y is



(72 + 8 + 35) + 14

= 129gm (molecular weight of organic salt)

So to get 1.29 gm organic salt.

We have to form 0.01 mole salt.

So 0.01 mole nitrobenzene is required.

0.03 mole Sn is required.

So the amount of nitrobenzene = 0.01 × 123 = 1.23 gm

the amount of Sn required = 0.01 × 357 = 3.57 gm

Ans. 3.57 & 1.23

Question Stem for Question Nos. 11 and 12

Question Stem

A sample (5.6 g) containing iron is completely dissolved in cold dilute HCl to prepare a 250 mL of solution. Titration of 25.0 mL of this solution requires 12.5 mL of 0.03 M $KMnO_4$ solution to reach the end point. Number of moles of Fe^{2+} present in 250 mL solution is x × 10^{-2} (consider complete dissolution of $FeCl_2$). The amount of iron present in the sample of y% by weight.

(Assume : $KMnO_4$ reacts only with Fe^{2+} in the solution

Use : Molar mass of iron as 56 g mol^{-1})

11. The value of x is ______.

Ans. (1.87 or 1.88)

Sol. $Fe + 2HCl \longrightarrow FeCl_2 + H_2$

x mole x mole

$Fe^{+2} + MnO_4^-$

$\frac{x}{10 \text{ mole}}$ 12.5 ml

$n_f = 1$ $n_f = 5$

$$\frac{x}{10} = \frac{12.5 \times 0.03 \times 5}{1000}$$

x = 0.01875 (x = 1.88 or 1.87)

wt of Fe = 1.05g

$$\%Fe = \frac{1.05}{5.6} \times 100 = 18.75$$

12. The value of y is ______.

Ans. (18.75)

Sol. $Fe + 2HCl \longrightarrow FeCl_2 + H_2$

x mole x mole

$Fe^{+2} + MnO_4^-$

$\frac{x}{10\text{ mole}}$ 12.5 ml

0.03 M

$n_f = 1$ $n_f = 5$

$$\frac{x}{10} = \frac{12.5 \times 0.03 \times 5}{1000}$$

x = 0.01875 (x = 1.88 or 1.87)

wt of Fe = 1.05g

$$\%Fe = \frac{1.05}{5.6} \times 100 = 18.75$$

SECTION-3 : (Maximum Marks : 12)

- This section contains **TWO (02) paragraphs**. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

Paragraph

The amount of energy required to break a bond is same as the amount of energy released when the same bond is formed. In gaseous state, the energy required for homolytic cleavage of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by s-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below :

$H_3C - H(g) \longrightarrow H_3C^{\bullet}(g) + H^{\bullet}(g) \quad \Delta H^\circ = 105 \text{ kcal mol}^{-1}$

$Cl - Cl(g) \longrightarrow Cl^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 58 \text{ kcal mol}^{-1}$

$H_3C - Cl(g) \longrightarrow H_3C^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 85 \text{ kcal mol}^{-1}$

$H - Cl(g) \longrightarrow H^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 103 \text{ kcal mol}^{-1}$

13. Correct match of the C–H bonds (shown in bold) in Column J with their BDE in Column K is

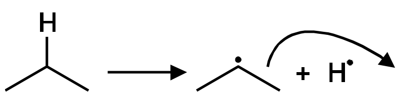
Column J **Molecule**	**Column K** **BDE (kcal mol^{-1})**
(P) $H–CH(CH_3)_2$	(i) 132
(Q) $H–CH_2Ph$	(ii) 110
(R) $H–CH=CH_2$	(iii) 95
(S) $H–C{\equiv}CH$	(iv) 88

(A) P – iii, Q – iv, R – ii, S – I (B) P – i, Q – ii, R – iii, S – iv

(C) P – iii, Q – ii, R –i, S – iv (D) P – ii, Q – i, R – iv, S – iii

Ans. (A)

Sol. Most stability of radical, less is the bond energy

(P)  2° Carbon Free radical

(Q) $Ph-CH_2-H \longrightarrow Ph-\dot{C}H_2 + H^{\bullet}$ Most stable due to resonance

(R) $CH_2=CH-H \longrightarrow CH_2=\dot{C}H + H^{\bullet}$ (less stable)

(S) $CH\equiv C-H \longrightarrow CH\equiv C^{\bullet} + H^{\bullet}$ More % S-Character Decrease stability of free radical

Q require least BDE and S Required maximum BDE

Max BDE

So, Order of BDE Q < P < R < S

14. For the following reaction

$$CH_4(g) + Cl_2(g) \xrightarrow{\text{light}} CH_3Cl(g) + HCl(g)$$

the correct statement is

(A) Initiation step is exothermic with $\Delta H° = -58$ kcal mol^{-1}

(B) Propagation step involving $^{\bullet}CH_3$ formation is exothermic with $\Delta H° = -2$ kcal mol^{-1}.

(C) Propagation step involving CH_3Cl formation is endothermic with $\Delta H° = +27$ kcal mol^{-1}.

(D) The reaction is exothermic with $\Delta H° = -25$ kcal mol^{-1}.

Ans. (D)

Sol. Initiation step is endothermic hence option (A) is wrong.

Propagation step involving $^{\bullet}CH_3$ formation is endothermic hence option (B) is wrong.

Propagation step involving CH_3Cl formation is exothermic hence option (C) is wrong.

Reaction

$CH_4 + Cl_2 \longrightarrow CH_3-Cl + HCl$

$CH_4 \longrightarrow CH_3^{\bullet} + H$ $\quad \Delta H = 105$ KCal / mol

$Cl_2 \longrightarrow Cl^{\bullet} + Cl^{\bullet}$ $\quad \Delta H = 105$ KCal / mol

$Cl^{\bullet} + CH_3^{\bullet} \longrightarrow CH_3-Cl$ $\quad \Delta H = 105$ KCal / mol

$Cl^{\bullet} + H^{\bullet} \longrightarrow HCl$ $\quad \Delta H = 105$ KCal / mol

$CH_4 + Cl_2 \longrightarrow CH_3-Cl + HCl$ $\quad \Delta H = -25$ KCal / mol

Overall reaction is exothermic with $\Delta H° = -25$ KCal/mol, hence option (D) is correct.

Paragraph

The reaction of $K_3[Fe(CN)_6]$ with freshly prepared $FeSO_4$ solution produces a dark blue precipitate called Turnbull's blue. Reaction of $K_4[Fe(CN)_6]$ with the $FeSO_4$ solution in complete absence of air produces a white precipitate X, which turns blue in air. Mixing the $FeSO_4$ solution with $NaNO_3$, followed by a slow addition of concentrated H_2SO_4 through the side of the test tube produces a brown ring.

15. Precipitate X is

(A) $Fe_4[Fe(CN)_6]_3$ (B) $Fe[Fe(CN)_6]$ (C) $K_2Fe[Fe(CN)_6]$ (D) $KFe[Fe(CN)_6]$

Ans. (C)

Sol. $K_4[Fe(CN)_6] \xrightarrow[\text{absence of air}]{FeSO_4} K_4Fe[Fe(CN)_6]$ White precipitate

$\downarrow$ air

$Fe_4[Fe(CN)_6]_3$
(Prussian Blue)

16. Among the following, the brown ring is due to the formation of

(A) $[Fe(NO)_2(SO_4)_2]^{2-}$ (B) $[Fe(NO)_2(H_2O)_4]^{3+}$ (C) $[Fe(NO)_4(SO_4)_2]$ (D) $[Fe(NO)(H_2O)_5]^{2+}$

Ans. (D)

Sol. $FeSO_4 \xrightarrow[\text{slow addition of conc. } H_2SO_4]{NaNO_3} \left[Fe(H_2O)_5NO\right]SO_4$
(Brown Ring Complex)

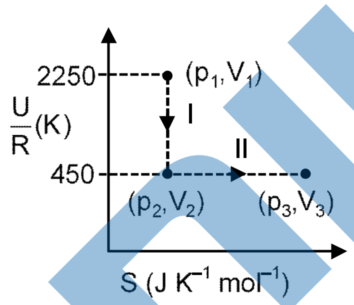
SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. One mole of an ideal gas at 900 K, undergoes two reversible processes, I followed by II, as shown below. If the work done by the gas in the two processes are same, the value of $\ln \frac{V_3}{V_2}$ is ___.



(U: internal energy, S: entropy, p: pressure, V: volume, R: gas constant)

(Given: molar heat capacity at constant volume, $C_{V,m}$ of the gas is $\frac{5}{2}R$)

Ans. (10)

Sol. $\Delta U_I = nC_{v,m}\,\Delta T = W_I\ \{q_I = 0\}$

$$-1800\,R = 1\times\frac{5R}{2}\times\Delta T = \Delta T = -720\,K$$

T_2 = 180 K

$$W_{II} = W_I = -1800\,R = -1\times R\times 180\ \ln\left(\frac{V_3}{V_2}\right)$$

$$\ln\left(\frac{V_3}{V_2}\right) = 10 \Rightarrow 10$$

18. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in cm s^{-1}) of He atom after the photon absorption is ___.

(Assume: Momentum is conserved when photon is absorbed.

Use: Planck constant = 6.6×10^{-34} J s, Avogadro number = 6×10^{23} mol^{-1}, Molar mass of He = 4 g mol^{-1})

Ans. (30)

Sol. $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{6.6 \times 10^{-34}}{330 \times 10^{-9}} = \dfrac{4 \times 10^{-3}}{6 \times 10^{23}} \times v$ (p = m × v)

v = 0.3 m/s = 30 cm/s

19. Ozonolysis of ClO_2 produces an oxide of chlorine. The average oxidation state of chlorine in this oxide is ___.

Ans. (6)

Sol. $2ClO_2 + 2O_3 \longrightarrow Cl_2O_6 + 2O_2$

Cl_2O_6

2x + 6(–2) = 0

x = +6

Average oxidation state of Cl in Cl_2O_6 is 6.

PART C : MATHEMATICS

SECTION-1 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If unanswered;
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

choosing any other option(s) will get −2 marks.

1. Let

$S_1 = \{(i, j, k) : i, j, k \in \{1, 2,......,10\}\}$

$S_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2,......,10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1, 2,......,10\}\}$.

and

$S_4 = \{(i, j, k, l) : i, j, k \text{ and } l \text{ are distinct elements in } \{1, 2,......,10\}\}$.

If the total number of elements in the set S_r is n_r, r = 1, 2, 3, 4, then which of the following statements is (are) TRUE?

(A) $n_1 = 1000$ (B) $n_2 = 44$ (C) $n_3 = 220$ (D) $\frac{n_4}{12} = 420$

Ans. (A, B, D)

Sol. (A) $n_1 = 10 \times 10 \times 10 = 1000$

(B) As per given condition $1 \le i < j + 2 \le 10 \Rightarrow j \le 8$ & $i \ge 1$

for i = 1, 2, $j = 1, 2, 3, ..., 8 \to (8 + 8)$ possibilities

for i = 3, $j = 2, 3, ..., 8 \to 7$ possibilities

i = 4, $j = 3, ..., 8 \to 6$ possibilities

i = 9, $j = 1 \to 1$ possibility

So $n_2 = (1 + 2 + 3 + ...+ 8) + 8 = 44$

(C) $n_3 = {}^{10}C_4$ (Choose any four)

$= 210$

(D) $n_4 = {}^{10}C_4 . 4! = (210)(24)$

$\Rightarrow \frac{n_4}{12} = 420$

So correct Ans. (A), (B), (D)

2. Consider a triangle PQR having sides of lengths p, q and r opposite to the angles P, Q and R, respectively. Then which of the following statements is (are) TRUE?

(A) $\cos P \ge 1 - \frac{p^2}{2qr}$

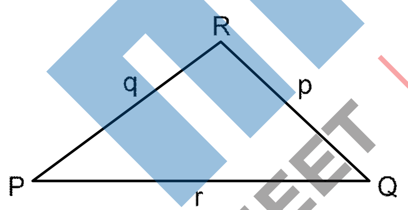
(B) $\cos R \ge \left(\frac{q-r}{p+q}\right)\cos P + \left(\frac{p-r}{p+q}\right)\cos Q$

(C) $\frac{q+r}{p} < 2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

(D) If $p < q$ and $p < r$, then $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

Ans. (A, B)

Sol.



(A) $\cos P = \frac{q^2 + r^2 - p^2}{2qr} = \frac{q^2 + r^2}{2qr} - \frac{p^2}{2qr} \ge 1 - \frac{p^2}{2qr}$

(as $p^2 + q^2 \ge 2qr$ (AM $\ge$ GM)), so (A) is correct

(B) $(p + q) \cos R \ge (q - r) \cos P + (p - r) \cos Q$

$\Rightarrow (p \cos R + r \cos P) + (q \cos R + r \cos Q) \ge q \cos P + p \cos Q$

$\Rightarrow q + p \ge r$

So (B) is correct

(C) $\frac{q+r}{p} = \frac{\sin Q + \sin R}{\sin P} \geq \frac{2\sqrt{\sin Q \times \sin R}}{\sin P}$ so (C) is incorrect

(D) $\cos Q > \frac{p}{r}$ $\Rightarrow$ sinR cosQ > sinP

$\Rightarrow$ sinP + sin (R – Q) > 2 sinP

$\Rightarrow$ sin (R – Q) > sinP

need not necessarily hold true if R < Q

Hence (A), (B)

3. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that

$$f(0) = 1 \text{ and } \int_0^{\frac{\pi}{3}} f(t)\, dt = 0$$

Then which of the following statements is (are) TRUE?

(A) The equation f (x) – 3 cos 3x = 0 has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(B) The equation f (x) – 3 sin 3x $= -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(C) $\lim_{x\to 0} \frac{x\int_0^x f(t)\, dt}{1-e^{x^2}} = -1$

(D) $\lim_{x\to 0} \frac{\sin x\int_0^x f(t)\, dt}{x^2} = -1$

Ans. (A, B, C)

Sol. (A) Let g(x) = f(x) – 3 cos 3x

Now $\int_0^{\pi/3} g(x)dx = \int_0^{\pi/3} f(x)dx - 3\int_0^{\pi/3} \cos 3x\, dx = 0$

Hence g(x) = 0 has a root in $\left(0, \frac{\pi}{3}\right)$

(B) Let $h(x) = f(x) - 3\sin 3x + \frac{6}{\pi}$

Now $\int_0^{\pi/3} h(x)dx = \int_0^{\pi/3} f(x)dx - 3\int_0^{\pi/3} \sin 3x\, dx + \int_0^{\pi/3} \frac{6}{\pi} dx$

= 0 – 2 + 2 = 0

Hence h(x) = 0 has a root in $\left(0, \frac{\pi}{3}\right)$

$$\text{(C)} \lim_{x\to 0} \frac{x\int_0^x f(t)dt}{1-e^{x^2}} = \lim_{x\to 0} \underbrace{\left(\frac{x^2}{1-e^{x^2}}\right)}_{-1} \underbrace{\frac{\int_0^x f(t)dt}{x}}_{\text{Apply L' Hopital's Rule}}$$

$$= -1\lim_{x\to 0}\frac{f(x)}{1} = -1$$

$$\text{(D)} \lim_{x\to 0}\frac{(\sin x)\int_0^x f(t)\,dt}{x^2}$$

$$= \lim_{x\to 0} \underbrace{\left(\frac{\sin x}{x}\right)}_{1} \underbrace{\frac{\int_0^x f(t)dt}{x}}_{\text{Apply L' Hopitals Rule}}$$

$$= 1\lim_{x\to 0}\frac{f(x)}{1} = 1$$

Ans. (A, B, C)

4. For any real numbers α and β, let $y_{\alpha,\beta}(x)$, $x \in \mathbb{R}$, be the solution of the differential equation

$$\frac{dy}{dx} + \alpha y = xe^{\beta x},\ y(1) = 1$$

Let S = $\{y_{\alpha,\beta}(x) : a, b \in \mathbb{R}\}$. Then which of the following functions belong(s) to the set S?

(A) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

(B) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

(C) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

(D) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

Ans. (A, C)

Sol. Integrating factor = $e^{\alpha x}$

So $ye^{\alpha x} = \int xe^{(\alpha+\beta)x}dx$

Case-I

If $\alpha + \beta = 0$ $\quad ye^{\alpha x} = \frac{x^2}{2} + c$

It passes through (1, 1) $\Rightarrow C = e^{\alpha} - \frac{1}{2}$

So $ye^{\alpha x} = \frac{x^2 - 1}{2} + e^{\alpha}$

for α = 1

$$y = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x} \to (A)$$

Case-II

If $\alpha + \beta \neq 0$

$$ye^{\alpha x} = \frac{x \cdot e^{(\alpha+\beta)x}}{\alpha+\beta} - \frac{1}{\alpha+\beta} e^{(\alpha+\beta)x}\, dx$$

$$\Rightarrow ye^{\alpha x} = \frac{x \cdot e^{(\alpha+\beta)x}}{\alpha+\beta} - \frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2} + c$$

$$\Rightarrow \text{So } c = e^{\alpha} - \frac{e^{\alpha+\beta}}{\alpha+\beta} + \frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}$$

$$y = \frac{e^{\beta x}}{(\alpha+\beta)^2}\left((\alpha+\beta)x - 1\right) + e^{-\alpha x}\left(e^{x} - \frac{e^{\alpha+\beta}}{\alpha+\beta} + \frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}\right)$$

If $\alpha = \beta = 1$

$$y = \frac{e^{x}}{4}(2x-1) + e^{-x}\left(e - \frac{e^2}{2} + \frac{e^2}{4}\right)$$

$$y = \frac{e^{x}}{4}\left(x - \frac{1}{2}\right) + e^{-x}\left(e - \frac{e^2}{4}\right) \to (c)$$

Ans. (A) & (C)

5. Let O be the origin and $\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OB} - \lambda\overrightarrow{OA})$ for some $\lambda > 0$. If $\left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \frac{9}{2}$, then which of the following statements is (are) TRUE?

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB is $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

Ans. (A, B, C)

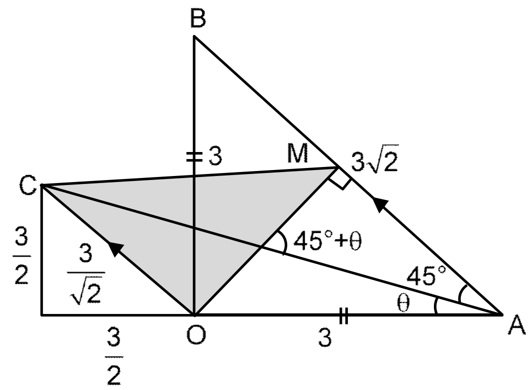
Sol. $\overrightarrow{OB} \times \overrightarrow{OC} = \frac{1}{2}\overrightarrow{OB} \times \left(\overrightarrow{OB} - \lambda\overrightarrow{OA}\right)$

$$= \frac{\lambda}{2}\left(\overrightarrow{OA} \times \overrightarrow{OB}\right)$$

$\left|\overrightarrow{OB}\right| \times \left|\overrightarrow{OC}\right| = \frac{\lambda}{2}\left|\overrightarrow{OA}\right| \times \left|\overrightarrow{OB}\right|$ (Note $\overrightarrow{OA}$ & $\overrightarrow{OB}$ are perpendicular)

$\Rightarrow \frac{9\lambda}{2} = \frac{9}{2} \Rightarrow \lambda = 1$ (given $\lambda > 0$)

So $\overrightarrow{OC} = \frac{\overrightarrow{OB} - \overrightarrow{OA}}{2} = \frac{\overrightarrow{AB}}{2}$



M is mid point of AB

Note projection of $\overrightarrow{OC}$ on $\overrightarrow{OA} = -\frac{3}{2}$

$\tan\theta = \frac{1}{3}$

Area of $\Delta ABC = \frac{9}{2}$

Acute angle between diagonals is

$$\tan^{-1}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) = \tan^{-1} 2$$

6. Let E denote the parabola $y^2 = 8x$. Let P = (–2, 4), and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE?

(A) The triangle PFQ is a right-angled triangle

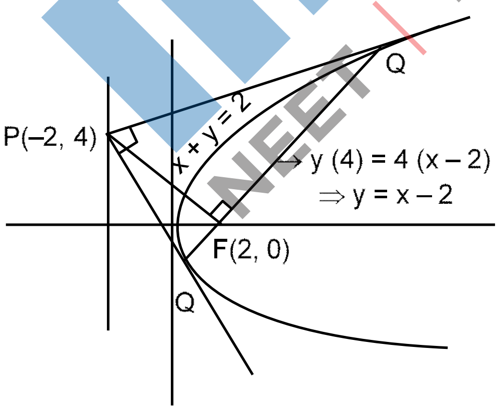
(B) The triangle QPQ' is a right-angled triangle

(C) The distance between P and F is $5\sqrt{2}$

(D) F lies on the line joining Q and Q'

Ans. (A, B, D)

Sol.



Note that P lies on directrix so triangle PQQ' is right angled, hence QQ' passes through focus F.

$PF = 4\sqrt{2}$

Equation of QF is y = x – 2 & PF is x + y = 2. Hence A, B, D.

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

  Zero Marks : 0 In all other cases.

Question Stem for Questions Nos. 7 and 8

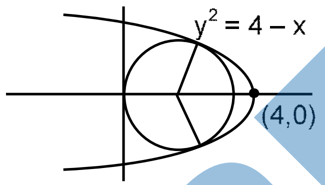
Question Stem

Consider the region $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

7. The radius of the circle C is _____.

Ans. (1.50)

Sol.



Let the circle be

$x^2 + y^2 + \lambda x = 0$

For point of intersection of circle & parabola $y^2 = 4 - x$.

$x^2 + 4 - x + \lambda x = 0 \Rightarrow x^2 + x(\lambda - 1) + 4 = 0$

For tangency : $\Delta = 0 \Rightarrow (\lambda - 1)^2 - 16 = 0 \Rightarrow \lambda = 5$ (rejected) or $\lambda = -3$

Circle : $x^2 + y^2 - 3x = 0$

$\text{Radius} = \frac{3}{2} = 1.5$

8. The value of α is _____.

Ans. (2.00)

Sol. For point of intersection :

$x^2 - 4x + 4 = 0 \Rightarrow x = 2$ so $\alpha = 2$

Question Stem for Questions Nos. 9 and 10

Question Stem

Let $f_1 : (0, \infty) \to \mathbb{R}$ and $f_2 : (0, \infty) \to \mathbb{R}$ be defined by

$$f_1(x) = \int_0^x \prod_{j=1}^{21} (t-j)^j \, dt, \; x > 0$$

and

$$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, \; x > 0,$$

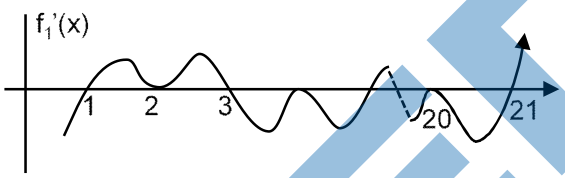
where, for any positive integer n and real numbers $a_1, a_2, \ldots\ldots, a_n$, $\Pi_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots\ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, i = 1, 2, in the interval $(0, \infty)$

9. The value of $2m_1 + 3n_1 + m_1n_1$ is _____.

Ans. (57.00)

Sol. $f_1(x) = \int_0^x \prod_{j=1}^{21} (t-j)^j \, dt$

$$f_1'(x) = \prod_{j=1}^{21} (t-j)^j = (x-1)(x-2)^2(x-3)^2 \ldots\ldots (x-21)^{21}$$



So points of minima one 4m + 1 where m = 0, 1,........5 $\Rightarrow m_1 = 6$

Points of maxima are 4m – 1 where m = 1, 2,........5 $\Rightarrow n_1 = 5$

$\Rightarrow 2m_1 + 3n_1 + m_1n_1 = 57$

10. The value of $6m_2 + 4n_2 + 8m_2n_2$ is _____.

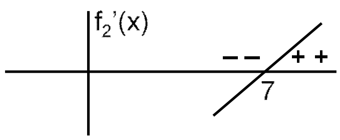
Ans. (6.00)

Sol. $f_2'(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450$

$\Rightarrow f_2'(x) = 2 \times 49 \times 50(x-1)^{49} - 50 \times 12 \times 49(x-1)^{48}$

$= 50 \times 49 \times 2(x-1)^{48}(x-1-6)$

$= 50 \times 49 \times 2\,(x-1)^{48}\,(x-7)$



Point of minima = 7

$\Rightarrow m_2 = 1$

No point of maxima

$\Rightarrow n_2 = 0$

$6m_2 + 4n_2 + 8m_2n_2 = 6$

Question Stem for Questions Nos. 11 and 12

Question Stem

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$, i = 1, 2, and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$ be functions such that

$g_1(x) = 1$, $g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x)\cdot g_i(x)\,dx$, i = 1, 2

11. The value of $\frac{16S_1}{\pi}$ is _____.

Ans. (2.00)

Sol. $S_1 = \int_{\pi/8}^{3\pi} f(x)dx = \int_{\pi/8}^{3\pi/8} \sin^2 x\,dx = \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{8} + \frac{3\pi}{8} - x\right)dx = \int_{\pi/8}^{3\pi/8} \cos^2 x\,dx$

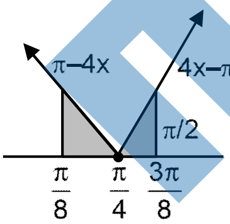
$$2S_1 = \int_{\pi/8}^{3\pi/8} (\sin^2 x + \cos^2 x)dx = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{\pi}{4}$$

$$\Rightarrow \frac{16S_1}{\pi} = 2$$

12. The value of $\frac{48S_2}{\pi^2}$ is _____.

Ans. (1.50)

Sol.



$$S_2 = \int_{\pi/8}^{3\pi} f(x)\cdot g_2(x)dx = \int_{\pi/8}^{3\pi/8} \sin^2 x|4x - \pi|dx$$

$$= \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{2} - x\right)\left|4\left(\frac{\pi}{2} - x\right) - \pi\right|dx \qquad = \int_{\pi/8}^{3\pi/8} \cos^2 x|\pi - 4x|dx$$

$$\Rightarrow 2S_2 = \int_{\pi/8}^{3\pi/8} |4x - \pi|(\sin^2 x + \cos^2 x)dx = \int_{\pi/8}^{3\pi/8} |4x - \pi|dx =$$

$$= 2 \times \frac{1}{2} \times \frac{\pi}{8} \times \frac{\pi}{2} = \frac{\pi^2}{16} \qquad \Rightarrow \frac{48S_2}{\pi^2} = \frac{3}{2} = 1.5$$

SECTION-3 : (Maximum Marks : 12)

- This section contains **TWO (02) paragraphs**. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3 If ONLY the correct option is chosen;
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1 In all other cases.

Paragraph

Let

$$M = \{(x,y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq r^2\},$$

where r > 0. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, n = 1, 2, 3, ... Let $S_0 = 0$ and, for $n \geq 1$, let S_n denote the sum of the first n terms of this progression. For $n \geq 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

13. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2\ell = 22$ (B) $2k + \ell = 26$ (C) $2k + 3\ell = 34$ (D) $3k + 2\ell = 40$

Ans. (D)

Sol. $S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{n-1}}$

$= 2\left(1 - \frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$

Centre of C_n is $\left(2 - \frac{1}{2^{n-2}}, 0\right)$

and radius of C_n is $\frac{1}{2^{n-1}}$

when $r = \frac{1025}{S13} < 2$

C_n will lie inside m

when $2 - \frac{1}{2^{n-2}} + \frac{1}{2^{n-1}} < \frac{1025}{S13}$

$\Rightarrow k = 10$

Also $\ell = 5$

$3k + 2\ell = 30 + 10 = 40$

Ans. (D)

14. Consider M with $r = \dfrac{(2^{199}-1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

Ans. (B)

Sol. Center of D_n is (S_{n-1}, S_{n-1})

$r = \dfrac{1}{2^{n-1}}$

D_n will lie inside

when $\sqrt{2}\left(S_{n-1}\right) < \dfrac{2^{199}-1}{2^{198}}\sqrt{2}$

$\Rightarrow \dfrac{\sqrt{2}}{2^{n-2}} > \dfrac{\sqrt{2}}{2^{198}} + \dfrac{1}{2^{n-1}}$

$\Rightarrow n = 199$

Paragraph

Let $\psi_1 : [0, \infty) \to$ □, $\psi_2 : [0, \infty) \to$ □, $f : [0, \infty) \to$ □ and $g : [0, \infty) \to$ □ be functions such that

$f(0) = g(0) = 0,$

$\psi_1(x) = e^{-x} + x, x \geq 0,$

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0,$

$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}\,dt, \; x > 0$

and

$g(x) = \int_0^{x^2} \sqrt{t}\, e^{-t^2}\,dt, \; x > 0$

15. Which of the following statements is TRUE ?

(A) $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \dfrac{1}{3}$

(B) For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

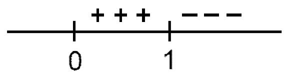
(C) For every $x > 0$, there exists $\alpha\ \beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi 1(\beta) - 1)$

(D) f is an increasing function on the interval $\left[0, \dfrac{3}{2}\right]$

Ans. (C)

Sol. $f'(x) = \left(|x| - x^2\right)e^{-x^2} + \left(|x| - x^2\right)e^{-x^2}, \; x \geq 0$

$f'(x) = 2\left(x - x^2\right)e^{-x^2}$



hence option (D) is wrong

$g'(x) = xe^{-x^2} 2x$

$f'(x) + g'(x) = 2 \times e^{-x^2}$

$f(x) + g(x) = -e^{-x^2} + c$

$f(x) + g(x) = -e^{-x^2} + 1$

$f(\ln 3) + g(\sqrt{\ln 3}) = 1 - \frac{1}{3} = \frac{2}{3}$ (option (A) is wrong)

$H(x) = \psi_1(x) - 1 - \alpha x = e^{-x} + x - 1 - \alpha x$, $\quad x \geq 1$ & $\alpha \in (1, x)$

$H(1) = e^{-1} + 1 - 1 - \alpha < 0$

$H'(x) = -e^{-x} + 1 - \alpha > 0 \Rightarrow H(x)$ is $\downarrow \Rightarrow$ option (B) is wrong

(C) $\psi_2(x) = 2(\psi_1(\beta) - 1)$

Applying L.M.V.T to $\psi_2(x)$ in $[0, x]$

$\psi_2'(\beta) = \frac{\psi_2(x) - \psi_2(0)}{x}$

$2\beta - 2 + 2e^{-\beta} = \frac{\psi_2(x) - 0}{x}$

$\Rightarrow \psi_2(x) = 2x(\psi_1(\beta - 1))$ has one solution

option (C) is correct.

16. Which of the following statements is TRUE ?

(A) $\psi_1(x) \leq 1$, for all $x > 0$

(B) $\psi_2(x) \leq 0$, for all $x > 0$

(C) $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(D) $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Ans. (D)

Sol. (A) $\psi_1(x) = e^{-x} + x,\ x \geq 0$

$\psi_1'(x) = 1 - e^{-x} > 0 \quad \Rightarrow \psi_1(x)$ is $\uparrow$

$\psi_1(x) \geq \psi_1(0) \quad \forall x \geq 0 \Rightarrow \psi_1(x) \geq 1$

(B) $\psi_2(x) = x^2 - 2x + 2 - 2e^{-x} \quad x \geq 0$

$\psi_2'(x) = 2x - 2 + 2e^{-x} = 2\psi_1(x) - 2 \geq 0 \quad \forall x \geq 0$

$\Rightarrow \psi_2(x)$ is $\uparrow \Rightarrow \psi_2(x) \geq \psi_2(0) \Rightarrow \psi_2(x) \geq 0$

(C) $f(x) = 2\int_0^x (t - t^2)e^{-t^2}\,dt \quad \& \qquad x \in \left(0, \frac{1}{2}\right)$

$= 2\int_0^x 2te^{-t^2}\,dt - \int_0^x 2t^2e^{-t^2}\,dt$

$= -e^{-x^2}\Big|_0^x$

Let $H(x) = f(x) - 1 + e^{-x^2} + \frac{2}{3}x^2 - \frac{2}{5}x^5, x \in \left(0, \frac{1}{2}\right)$

H (0) = 0

$H'(x) = 2(x - x^2)e^{-x^2} - 2xe^{-x^2} + 2x^2 - 2x^4$

$= -2x^2e^{-x^2} + 2x^2 - 2x^4$

$= 2x^2\left(1 - x^2 - e^{-x^2}\right)$

$\because\ e^{-x} \geq 1 - x\ \forall\ x \geq 0$

$\Rightarrow H'(x) \leq 0$

$\Rightarrow H(x)$ is $\downarrow \Rightarrow 1 - 1(x) < 0\ \forall x \in \left(0, \frac{1}{2}\right)$

Let $P(x) = g(x) - \frac{2}{3}x^3 + \frac{2}{5}x^5 - \frac{1}{7}x^7 \quad x \in \left(0, \frac{1}{2}\right)$

$P'(x) = 2x^2e^{-x^2} - 2x^2 + 2x^4 - x^6$

$$= 2x^2\left(1 - \frac{x^2}{1} + \frac{x^4}{2} - \frac{x^6}{3} + \ldots\right) - 2x^2 + 2x^4 - x^6$$

$$= -\frac{x^8}{3} + \frac{x^{10}}{12} \ldots\ldots$$

$\Rightarrow P'(x) \leq 0$

$\Rightarrow P(x)$ is $\downarrow$

$\Rightarrow P(x) \leq 0$

option (D) is correct

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

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17. A number is chosen at random from the set {1, 2, 3, ... , 2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ___.

Ans. (214)

Sol. A = set of numbers divisible by 3

A = {3, 6, 9, 12, ... 1998}

$\therefore$ n (A) = 666

B = set of numbers divisible by 7

B = {7, 14, 21, ...1995}

$\therefore$ n(B) = 285

$A \cap B$ = {21, 42,.......1995}

$\therefore$ $n(A \cup B) = 606 + 285 - 95 = 856$

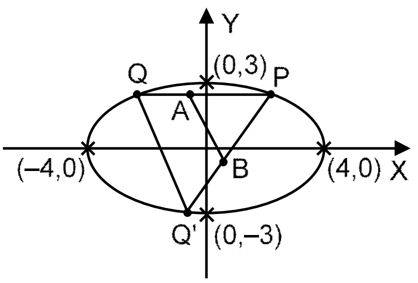
required probability $= \frac{856}{2000} = P$

so, $500\,P = \frac{856}{2000} \times 500 = 214$

18. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{19} = 1$. For any three distinct points P, Q and Q' on E, let M (P, Q) be the mid-point of the line segment joining P and Q, and M (P, Q') be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between M(P, Q) and M(P, Q'), as P, Q and Q' vary on E, is ____.

Ans. (4)

Sol.



A and B be midpoints of segment PQ and PQ' respectively

AB = distance between M(P, Q) and M(P, Q') $= \frac{1}{2} \cdot QQ'$

Since, Q, Q' must be on E, so,

maximum of QQ' = 8

$\therefore$ Maximum of $AB = \frac{8}{2} = 4$

19. For any real number x, let [x] denote the largest integer less than or equal to x. If

$$I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx,$$

then the value of 9I is ____.

Ans. (182)

Sol. Let $f(x) = \left(\frac{10x}{x+1}\right)$

So, $f'(x) = 10\left(\frac{(x+1)-x}{(x+1)^2}\right) = \frac{10}{(x+1)^2} > 0 \ \forall \ x \in [0,10],$

So, f(x) is an increasing function

So, range of f(x) is $\left[0, \sqrt{\frac{100}{11}}\right]$

$$I = \int_0^{1/9} \left[\sqrt{\frac{10x}{x+1}}\right] dx + \int_{2/3}^{9} \left[\sqrt{\frac{10x}{x+1}}\right] dx + \int_{1/9}^{2/3} \left[\sqrt{\frac{10x}{x+1}}\right] dx + \int_{2/3}^{9} \left[\sqrt{\frac{10x}{x+1}}\right] dx + \int_{9}^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$$

$$= 0 + \int_{1/9}^{2/3} dx + 2\int_{2/3}^{9} dx + 3\int_{9}^{10} dx$$

$$= \frac{2}{3} - \frac{1}{9} + 2\left(9 - \frac{2}{3}\right) + 3(10-9)$$

$$= \frac{6-1}{9} + 2\frac{25}{3} + 3 = \frac{5}{9} + \frac{50}{3} + 3$$

$$= \frac{5+150+27}{9} = \frac{182}{9} = 182$$